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Hydrodynamic Forces on Some Float Forms

JOHN A. MERCIER*
Stevens Institute of Technology, Hoboken, N. J.

Results of hydrodynamic tests of several models of typical float shapes are presented. Measurements of lift, drag, and pitching moment are made for the heave, surge, and pitch modes of motion in otherwise smooth water and for the model held fixed in traveling surface waves. These results are necessary for evaluating the motions of these bodies for arbitrary mass distributions, using the equations of motions. Coefficients expressing the inertial and damping characteristics of these models, based on the assumption of linearity of forces with motion and wave amplitude are presented in tables. Amplitudes and phases for the wave exciting forces are tabulated. Models tested include a half-immersed sphere, a half-immersed torus, a one-fiftieth scale model of the "Monster Buoy," a shallow draft rectangular barge, and a cylinder with a square damping plate at the lower end and with a hemispherical bottom cap.

Nomenclature

$\bar{A}_{ij}$ = coefficient (amplitude and phase) expressing force in ith direction due to jth mode of motion, per unit amplitude motion [used in Eq. (1)]; lb/in., for example
A_{wp} = area of waterplane, in.^{2}
B_k = nondimensional coefficient expressing amplitude of kth component of force in the presence of wave [defined in Eqs. (A20–A22) and in Table 2]
F_i = force (or moment) imposed on body by the fluid, ith component [used in Eq. (6)]; lb, or in.-lb
f = frequency of motion, cycles/sec
g = acceleration of gravity, in./sec^2
h_{ij} = nondimensional damping coefficient expressing out-of-phase part of ith component of hydrodynamic force due to jth mode of motion [defined as in Eq. (3)]
I_{ij} = component of moment-of-inertia tensor of body, e.g., $I_{xy} = \int\int xydm$ [used in Eq. (6)]; in.-lb sec^2
I_{wp} = geometrical moment of inertia of waterplane about axis of angular motion of buoy, in.^{4}
k_{ij} = nondimensional added mass coefficient expressing inertial part of ith component of hydrodynamic force due to jth mode of motion [defined as in Eq. (2)]
= distance of wave sensor from the origin of coordinates
m = mass of body [used in Eq. (6)]; lb-sec^2/in.
r = significant buoy dimension, radius of hemisphere which has same displacement as buoy, $r = (3\nabla/2\pi)^{1/3}$, in.
t = time, sec
γ_k = phase between occurrence of maxima of kth component of wave-induced force and wave trough passage, deg
Δ = buoy displacement, lb
∇ = buoy displaced volume, in.^{3}
η = wave elevation, in.
λ = wavelength, in.
μ = "added mass" due to hydrodynamic pressure distribution [used in Eq. (A3), for example]; lb-sec^2/in.
ρ = fluid mass density, lb-sec^2/in.^{4}
ω = frequency of motion, rad/sec

***Coordinate systems (illustrated in Fig. 2)*†**

X_1,X_2,X_3 = coordinate system fixed in inertial reference frame
X_1',X_2',X_3' = coordinate system parallel to X_1, X_2, X_3 with origin at center of still-water plane of model
X,Y,Z = coordinate system fixed in body with origin at center of still-water plane of model
$\xi_1,\xi_2,\xi_3,\varphi,\theta,\psi$ = surge, sway, heave, roll, pitch, and yaw motions

***Subscripts*‡**

am = associated with "added mass"
buoy = associated with buoyant forces
d = associated with disturbance (wave)
in-phase = relative to motion
m = associated with motion
o = indicating maximum value for wave elevation
out-of-phase = relative to motion
tare = associated with a necessary experimental correction

Introduction

A PROGRAM of research on the use of floats to support air-sea craft above the sea surface in such a way as to minimize the craft's motion in waves has been underway at

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* Research Engineer, Davidson Laboratory.

† A coordinate direction is sometimes used as a name of a force component [as in Eqs. (1, 5, A1, etc.)].

‡ Coordinates are sometimes used as a means of identifying components of forces, perhaps associated with a mode of motion [as in Eqs. (1, 2, 5, etc.)].

Davidson Laboratory. Naval Air Systems Command is interested in the possibility of extending the performance capability of such craft by affording a comfortable and stable working platform in heavy seas.

Hydrodynamical aspects of the floats used to support such craft are of great importance in design of such sea-sitting systems, but, unfortunately, are not well understood or documented in technical literature. Consequently, the present work was undertaken to obtain basic information on several types of float configurations. Included in the investigation are a slender vertical float, a barge-like form, and several buoy-like forms. The analysis of motions of craft which do not have forward speed may, of course, be related for buoys, barges, and air-sea craft as envisioned here.

Some studies of the hydrodynamics and motions of sea-sitting float forms have been carried out in the course of development of the "Monster Buoys" for the ONR Ocean Data System by General Dynamics/Convair. This investigation, which has been briefly described by Devereux and Jennings,[1] simulated buoy performance in combinations of wind, waves, and currents using complete scale models with prescribed center-of-gravity location, gyradii, etc. A similar investigation of air-to-navigation buoys has been carried out for the Coast Guard by DeSaix[2] at Davidson Laboratory.

This report presents data on the forces which act on the float forms under two conditions: when the models are held rigid and surface waves pass by, and when the model is forced to oscillate in various modes of motion in otherwise smooth water. These experimental results may be used in obtaining solutions of the system of differential equations of motion for a float having any prescribed mass distribution qualities when acted upon by waves. The hydrodynamic forces are thus separated from the inertial characteristics. The influence of wind loads, currents, mooring cable forces, gyro forces, etc., on the motions must be considered in addition to the hydrodynamic effects of wave action, but these are not treated in this report.

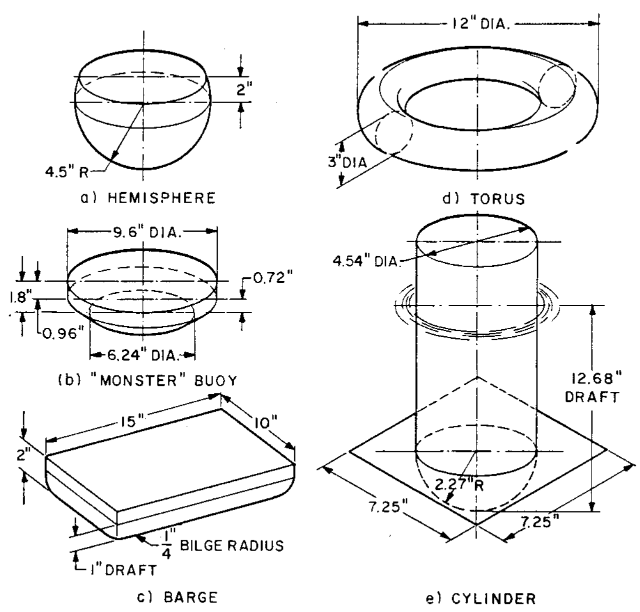


Fig. 1 Dimensions for float models tested.

Model	∇ (in.³)	r (in.)	Weight (lb)
a	191	4.5	1.11
b	33.7	2.525	0.565
c	149.3	4.15	1.16
d	100	3.63	0.72
e	193.3	4.51	1.27

Determination of the motions of floating structures in a seaway through the use of the equations of motion has been successfully utilized in ship theory for nearly twenty years. The techniques have been described by Korvin-Kroukovsky,[3] and others. The use of experimentally derived hydrodynamic forces to evaluate the motions of float-supported configurations was demonstrated by Mercier.[4] In that case it was possible to obtain reasonably good agreement between the measured motions of a free four-float platform and the motions computed using experimentally determined hydrodynamic forces.

Although no comparisons will be made between computed and measured motions for any of the float forms tested, a brief description of the method will be given. The forces due to waves are assumed to be independent of the forces due to float motion, which is probably valid for small motions. In this case the equations of motion may be linearized. The exciting forces due to waves are assumed to be proportional to the wave height and the forces due to the motion of the floats in smooth water are assumed proportional to the motions. The forces may be described by their amplitude and phase relative to the waves or motions, and are assumed to be frequency-dependent—but simple harmonic in time—for regular waves and simple harmonic motions. With these assumptions, the differential equations of motion reduce to algebraic equations (for regular waves and harmonic motions). To determine response to more realistic long-crested irregular seas, either deterministic or statistical descriptions of the vehicles behavior[5] may be used in conjunction with the transfer functions obtained for harmonic response. It is important to note that the response of vehicles in irregular seas may be derived from the linearized theory even for rather severe wave heights, for which the regular sea response would be quite nonlinear.[5] A brief presentation of the equations of motion for the floats, showing the hydrodynamic forces associated with waves and with motion in otherwise calm water in explicit form, will be given in the Analysis Section.

Five different hull configurations have been tested and results are presented here in the form of nondimensional coefficients which are dependent upon a frequency parameter. The coefficients and manner of presentation will be discussed in later sections of this paper. Sketches of the five models which have been tested are shown in Fig. 1. They include a half-immersed sphere, a half-immersed torus, a $\frac{1}{50}$ scale model of the "Monster Buoy,"[1] a rectangular barge form, and a cylinder with a square "damping" plate at the lower end of a hemispherical bottom cap. The last model has been used as one of four floats supporting a dynamic model of a platform and the test results have been presented previously although in a different form.[4]

Analysis

Hydrodynamic Forces

The hydrodynamic forces which act on the float model are determined with respect to a set of coordinate axes whose origin is fixed at the center of the still-water plane of the model. Oscillatory displacements of the model are also measured with respect to this axis system, shown in Fig. 2.

X_m, Z_m and Θ_m are hydrodynamic forces associated with the motion of the float in smooth water, determined from a series of tests with the float oscillated in a variety of combinations of pitch, surge, and heave motion, and described by the equations below.

$$X_m = \bar{A}_{xx}'x + \bar{A}_{xz}'z + \bar{A}_{x\theta}'\theta \quad (1a)$$

$$Z_m = \bar{A}_{zx}'x + \bar{A}_{zz}'z + \bar{A}_{z\theta}'\theta \quad (1b)$$

$$\Theta_m = \bar{A}_{\theta x}'x + \bar{A}_{\theta z}'z + \bar{A}_{\theta\theta}'\theta \quad (1c)$$

The coefficients $\bar{A}_{ij}'$ (amplitude and phase) of these equations will be determined for a number of values of frequency. The subscript m refers to the association of the force with the motion. It should be borne in mind that x, z, and θ, the surge, heave and pitch displacements, respectively, are not generally in phase with one another. Furthermore, for floats with longitudinal and transverse symmetry, a vertical motion should give rise to no surge or pitch force (for the origin of coordinate axes on the axis of symmetry); hence $\bar{A}_{xz}' = \bar{A}_{\theta z}' = 3$. Similar expressions may be written for sway, roll, or yaw motions, although most of the models, excepting the rectangular barge, are similar in the longitudinal and transverse directions. The sphere, torus, and monster buoy models are rotationally symmetric.

The forces due to the motions are assumed to be linear and superposable, as Eq. (1) indicates (the assumption can be checked by experiment). These forces can be expressed in terms of nondimensional coefficients which are assumed to depend on the frequency parameter $\omega^2 r/g$, where ω is the frequency of harmonic motion, r is a significant buoy dimension, in this case the radius of the hemisphere which has the same displacement as the buoy, and g is the acceleration of gravity. This form of frequency dependence of the forces is governed by the presence of the free-water surface.

The measured forces will be resolved into components representing static restoring forces due to buoyancy, which may be computed or obtained from static tests, "added mass" forces which are 180° out of phase with the buoyant forces, and damping forces which are 90° out of phase with both the static force and the added mass force. The damping force is associated with waves which radiate outward from the oscillating buoy and with viscous effects, which are expected to have the most significance for the cylindrical float model with the horizontal plate. The free surface influence on the added mass force is a manifestation of the standing wave system associated with the motion. The added mass is expressed in nondimensional form as a fraction of the displaced mass of the model, while the added moment of inertia is given as a fraction of the product of the displaced mass, $\rho\nabla$, and the radius of the equivalent hemisphere, so that, for example,

$$k_{zz} = \frac{\text{added mass force in } z\text{-direction due to } z\text{-motion}}{\omega^2 z \rho \nabla} \quad (2a)$$

$$k_{\theta x} = \frac{\text{added mass pitch moment due to } x\text{-motion}}{\omega^2 x r \rho \nabla} \quad (2b)$$

Linear and angular damping coefficients are expressed in a similar manner, e.g.,

$$h_{x\theta} = \frac{\text{out-of-phase force in } x\text{-direction due to } \theta\text{-motion}}{\omega^2 \theta r^2 \rho \nabla}$$

$$(3)$$

$$h_{\theta\theta} = \frac{\text{out-of-phase pitch moment due to } \theta\text{-motion}}{\omega^2 \theta r^2 \rho \nabla}$$

The forces (amplitude and phase) due to the wave disturbance on a single float are determined (see below) from an experiment on a restrained model in regular waves. These forces are proportional to wave height and are dependent on wave frequency.

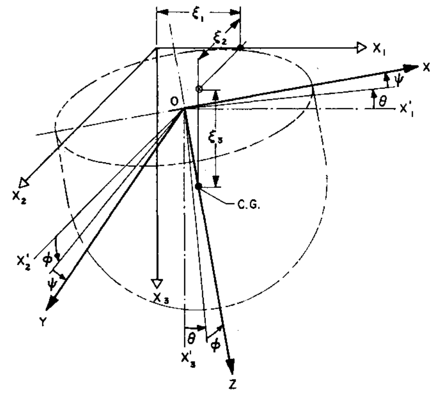
For a water wave of length λ progressing in the $-x$ direction and measured at the longitudinal location x, the wave elevation is given by

$$\eta = \eta_0 e^{i(\omega t + 2\pi x/\lambda)(c/l)} \quad (4)$$

The wave is measured by a sensor located at a distance l from the float, and the vertical component of the force which the wave induces on a float can be expressed as

$$Z_d = \Delta(\eta_0/r)\bar{B}_z(\omega^2 r/g)e^{i\omega t}\, e^{-i(\omega^2 r/g)(l/r)} \quad (5)$$

where $\bar{B}_z = B_z e^{i\gamma_z}$ is a dimensionless complex function of the frequency parameter. The subscript d refers to the association of the force with the disturbance. Thus, wave-induced forces acting on a float can be determined in terms of wave measurements obtained at some other location.



X, X₂ X₃ COORDINATE SYSTEM FIXED IN INERTIAL REFERENCE FRAME
X'₁ X'₂ X'₃ COORDINATE SYSTEM PARALLEL TO X, X₂ X₃
X Y Z COORDINATE SYSTEM FIXED IN BODY WITH ORIGIN AT CENTER OF STILL WATERPLANE OF MODEL
ξ_1 $\xi_2 \xi_3$ $\phi\ \theta\ \psi$: SURGE, SWAY, HEAVE, ROLL, PITCH AND YAW MOTIONS

Fig. 2 Coordinate systems.

Equations of Motion

The linearized equations of motion of a freely floating body undergoing small-amplitude oscillations in response to the seaway have been derived by Wong,[6] for a system of axes fixed at the center of gravity of the body. In order to use these equations with the coordinate system adopted for the present report, as shown in Fig. 2, some modifications are required, resulting in the following equations:

$$m\ddot{\xi}_1 = F_1 - \Delta\theta \quad (6a)$$

$$m\ddot{\xi}_2 = F_2 + \Delta\Phi \quad (6b)$$

$$m\ddot{\xi}_3 = F_3 + \Delta \quad (6c)$$

$$I_{11}\ddot{\Phi} + I_{12}\ddot{\theta} + I_{13}\ddot{\psi} = F_4 \quad (6d)$$

$$I_{21}\ddot{\Phi} + I_{22}\ddot{\theta} + I_{23}\ddot{\psi} = F_5 \quad (6e)$$

$$I_{31}\ddot{\Phi} + I_{32}\ddot{\theta} + I_{33}\ddot{\psi} = F_6 \quad (6f)$$

The quantities (ξ_1, ξ_2, ξ_3, Φ, θ, ψ) correspond to the modes of motion (surge, sway, heave, roll, pitch, yaw), respectively. The F_i's represent the forces and moments imposed on the body by the water and by gravity, and include the average force (buoyancy), forces due to waves, and forces due to body motion. The product of inertia terms in Eqs. (6d, e, and f) are, of course, symmetrical so that $I_{13} = I_{31}$, etc. For symmetry of the mass distribution about the longitudinal vertical plane x_1,x_2, two of these products vanish, viz., $I_{12} = I_{23} = 0$, while if symmetry also exists about the transverse vertical plane, x_2,x_3 all of the product terms are zero.

One of the above equations, the surge equation, will be expanded to demonstrate the manner in which the hydrodynamic forces are explicily incorporated. It should be recalled that the forces are determined in terms of the system of coordinates fixed in the body.

$$\begin{aligned} m\ddot{\xi}_1 &= X_d + X_m - \Delta\theta \quad (7a) \\ &= \Delta(\eta_0/r)\bar{B}_x e^{-i(\omega^2 r/g)(l/r)}\, e^{i\omega t} \\ &+ \Delta\theta e^{i\omega t} \text{ (static term)} \\ &+ i(\omega^2 \Delta e^{i\omega t}/g)(x h_{xx} + \theta t h_{x\theta}) \text{ (damping term)} \\ &- (\omega^2 \Delta e^{i\omega t}/g)(x k_{vx} + \theta r k_{x\theta}) \text{ (added mass)} \end{aligned}$$

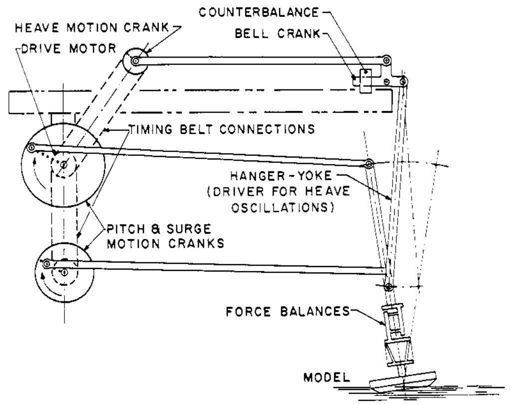


Fig. 3 Sketch of model oscillator.

$$- \Delta\theta e^{i\omega t} \text{ (gravity term)} \qquad (7b)$$

It has been assumed that there is no hydrodynamic coupling between surge force and heave, roll, sway, or yaw motion.

Equations for the coupled pitch-heave-surge oscillation of platforms supported by many rigidly connected floats have been given, in similar notation, by Mercier.[4] Expressions could also be written for analyzing the motions response of vehicles supported by floats which are pin-jointed, or otherwise nonrigidly connected, to the platform. These would be useful for design analysis of configurations such as the Nexus[7] platform or similar articulated, multi-float configurations.

Experiments

Test Techniques

Two methods are available for determining added mass and damping characteristics. The older method is the free oscillation technique used by Froude[8] and others,[3] in which the added mass and damping forces are inferred from the natural frequency and the curve of decaying oscillations. The forced oscillation technique has also been used extensively, as noted by Korvin-Kroukovsky,[3] in studies of oscillating ship forms, cylindrical sections, and other mathematically defined shapes. The advantages of the forced-oscillation technique include the more detailed record of the force and the true periodicity of the motion, which is only approximated in the free oscillation procedure. On the other hand, there are significant difficulties associated with obtaining reliable dynamic force measurements, and the analysis of the data is laborious.

For this investigation the forced oscillation technique was selected. The buoy models were attached to a motion generator through a system of balances and oscillated in a large tank of otherwise still water. The added mass and damping forces were subsequently derived from analysis of oscillograms of the forces and motions, corrections being applied based upon forced oscillation tests of the models in air.

Determinations of the wave-exciting forces were obtained from tests of the models held fixed in the large tank while regular waves passed by. Oscillograms of the forces and of a wave sensor located near the model were analyzed to determine the amplitude and phase of these forces.

Models

The shapes of the models tested are shown in Fig. 1. Dimensions, displaced volumes, and values for r, the radius of the hemisphere having the same displacement, are shown on this sketch. All models were made of low-density polyurethane foam coated with clear epoxy paint. The cylindrical float had a $\frac{1}{8}$-in.-thick horizontal plexiglass damping plate with sharply bevelled edges. The weights of the models, which must be kept light for the dynamic tests, are also shown in the figure.

Apparatus

The oscillatory forces acting on a single float were determined for two conditions: 1) with the model held fixed in regular waves; and 2) with the float oscillated in otherwise undisturbed water so as to produce combinations of pitch, heave, and surge motions. Roll and sway motions were also tested for the barge model. No tests were conducted with yaw motion.

A simple machine was devised to produce the required combinations of motions. A sketch of the oscillator is given in Fig. 3. The range of possible motions included heave, $\pm 1\frac{3}{8}$ in. max, in increments of $\frac{1}{8}$ in.; surge, ± 2 in. max, in increments of $\frac{1}{4}$ in.; and pitch, $\pm 10°$ max, in increments of 1°. The frequency of the motions was controlled by adjusting the speed of a geared d.c. motor. The model was connected to the machine's driven element through a system of balances which measured heave force, surge force, and pitch moment about a prescribed axis. For the sphere, torus, monster, and barge buoys, this pitch axis lay in the undisturbed water surface. For the cylindrical float, which had been tested previously,[4] the pitch axis was $6\frac{3}{8}$ in. below the water surface. Pitch moment results for this form have been corrected to apply to an axis system whose origin is at the water surface to be consistent with other results given in this paper. This same device supported the model firmly for the tests in waves.

Test results are in the form of oscillograms of the force signals, suitably amplified and filtered to reduce high-frequency noise content. The motion of the model was recorded by the output of a rotary variable differential transformer used as a position transducer. This record indicated the regularity of the motion. The phase between the force and the motion was determined from the record of a switch which opened at a single point during the motion cycle. For tests of the fixed model in waves, a resistance type of wave wire was located in line with the model, approximately midway between the model and the sidewall of the tank. Wave records give both the wave amplitude and the phase information which relates the force and the passage of the wave trough.

Most of the tests were conducted in Davidson Laboratory's Tank 2, which is 75 ft square and 4.5 ft deep. The model and apparatus were attached to the tank rail structure at a location about 35 ft from the wavemaker, which extends along one wall of the tank, and 20 ft from the tank sidewall. The cylinder model had been tested in Tank 3, which is 300 ft long, 12 ft wide, and 6 ft deep, with the model located 75 ft from the wavemaker. The models had no forward speed.

Test Procedure and Program

The apparatus was secured to the rail of the towing tank, and the force balances and wave wire were calibrated. Tests were conducted in regular waves of varying frequency with peak-to-peak amplitudes of approximately 2 in. Enough signal cycles were recorded to assure steady-state conditions and to allow averaging for more accurate analysis. Approximately, 5 min was allowed between tests in waves of different frequency, to allow the waves to decay and the water surface to become smooth.

For tests in smooth water, with forced oscillation of the model, the position transducer was calibrated to correlate motion and oscillogram signal. Motor speed was set and a series of tests was run with different modes of oscillation and with different amplitudes. An adequate number of signal cycles were recorded, as before. Tests with combinations of pitch and heave, and of surge and heave, were conducted to learn whether or not important interactions would occur. Frequency range was limited on the low-frequency end by the inability of the motor speed control to provide harmonic motion for low speeds; and was limited on the high-frequency end by forced vibrations of the oscillator in the sidewise (across the tank) direction.

Since we want to describe those hydrodynamic forces (acting on the float) which are due to wave impingement and due to forced oscillation on smooth water, certain of the measurements made in the towing tank must be corrected to account for inertial and other forces not associated with fluid mechanics. These corrections apply only to forces due to forced oscillation. For the heave and surge motions, the forces are, basically, purely inertial; but for the pitch motion, the forces were measured in body-fixed axes, and therefore a weight component of the correction for side force and pitch moment must be introduced. The corrections were based upon tests of the model and apparatus conducted in air.

Results

General

Sample tracings of the oscillograms for heaving oscillations are shown in Fig. 4. These illustrate the general quality of the recorded data for the several modes of motion, which is not precisely regular and must be "smoothed." A more complete discussion of the oscillogram records and their analysis is given in Ref. 9.

An example of the uncorrected data which results from analysis of the oscillograms is given in Fig. 5, where test spots are shown for heave force amplitude and phase for the heave motion of the hemisphere model. Also shown in this chart is the faired curve for the measured force in air, which must be accounted for in evaluating the added mass and damping forces. In general, the data of all of the tests show a similar degree of "scatter."

The added mass and damping coefficients for the model are presented in tables for the various modes of forced oscillations, as a function of the previously defined frequency parameter $\omega^2 r/g$. The specific definition of each of the coefficients is given in Tables 1 and 2. The relevant geometrical data for the models are completely described in the illustrations of Fig. 1. An estimate of the accuracy of the tabulated coefficients, based on the author's judgment of the accuracies with which the measurements and analysis could be made, is included below each column of coefficients.

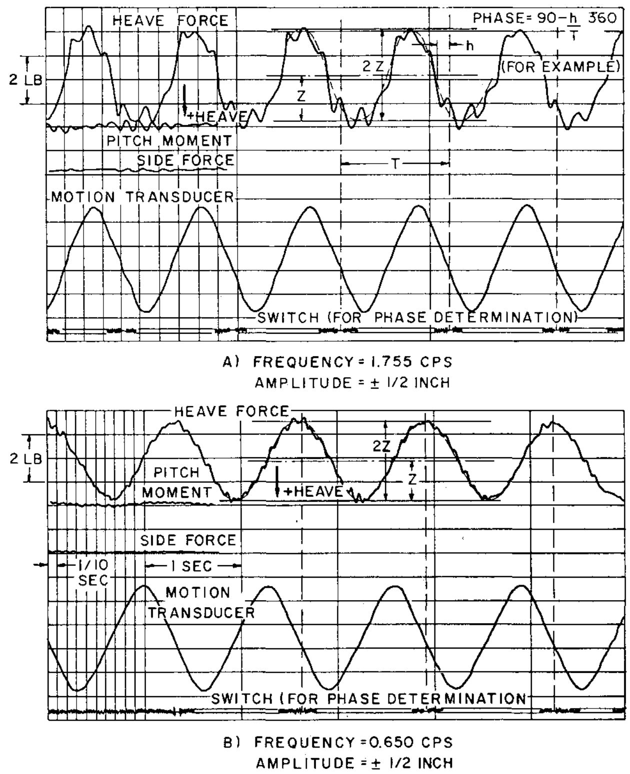


Fig. 4 Oscillogram records of forced heaving oscillations of barge model; switch opens at maximum positive heave.

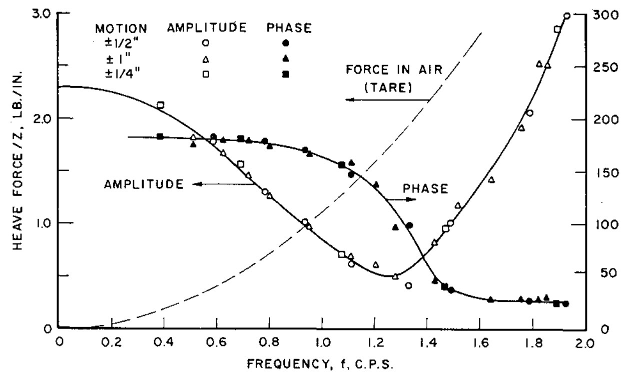


Fig. 5 Example of uncorrected data for heave force associated with heave motion.

Forced Heave Motion

For the models tested in this program, pure heave motions produce only vertical forces. These are due to buoyancy, inertial effects, and energy dissipation (damping). As has previously been mentioned in the Analysis Section, the inertial effect (or added mass force) is dependent on the free surface of the water because it is associated with a standing wave system. The damping force is associated with waves which radiate outward. Viscous effects on both the inertial and damping forces are also present, but are probably significant only in the case of the cylindrical float model with the horizontal plate.

The added mass and damping coefficients for this mode of motion are presented in Table 1a and 1b, respectively, as a function of the frequency parameter. Numerical values are comparable for a given frequency for the case of equal displacement of the various float forms. Further discussion and comparison of these data with other theoretical and experimental results will be given below.

Forced Horizontal Motion

Horizontal motion (surge or sway) produces, in general, a side force and a pitching moment which occur with frequency equal to the frequency of motion. The vertical force with the frequency of motion is too small to be measured, but a vertical force at twice the motion frequency is distinguishable. This is understandable since the force transverse to the direction of motion should be independent of the direction of the motion, velocity, or acceleration, for this case. The observance of this double-frequency force is interesting in itself, but no attempt has been made to correlate its magnitude or phase, since it is generally small compared to the other forces.

The half-immersed sphere does not experience any pitch moment about an axis through its center since all pressure forces acting on its surface are directed through the center. The friction forces are too small to measure.

The coefficients for this mode of motion are given in Table 1c–f; k_{xx} is the added mass coefficient for the x-directed inertial force, h_{xx} is the damping coefficient for the x-directed out-of-phase force, $k_{\theta x}$ is the added mass coefficient for the inertial pitch (or roll) moment, and $h_{\theta x}$ is the damping coefficient for the out-of-phase moment, all due to x-motion. A comparison of the data for the half-immersed sphere with theoretical results is discussed in the following section.

Forced Angular Motion

The pitch (or roll) mode of motion results in moments about the axis of motion and forces in the x (or y) direction

Table 1 Added mass and damping coefficients from forced oscillation tests

TABLE 1a

$k_{zz} = \frac{\text{added mass force in z-direction due to z-motion}}{\omega^2 z \rho \nabla}$

$\omega^2 r/g$	HEMISPHERE	TORUS	"MONSTER"	BARGE	CYLINDRICAL FLOAT
0.1	-	4.70	9.0	5.75	-
0.2	0.82	3.25	7.2	4.88	0.46
0.3	0.72	2.50	5.6	4.30	0.53
0.4	0.59	2.00	4.3	3.88	0.61
0.6	0.45	1.50	3.0	3.40	0.76
0.8	0.36	1.38	2.4	3.08	0.74
1.0	0.31	1.20	-	2.88	0.68
1.2	0.29	0.70		2.73	-
1.4	0.30	0.10		2.63	-
1.6	0.34	0.00	-	2.58	-
1.8	-	0.00	-	2.54	-
Estimated Accuracy	±0.1	±0.25	±0.5	+0.1	+0.1

TABLE 1b

$h_{zz} = \frac{\text{out-of-phase force in z-direction due to z-motion}}{\omega^2 z \rho \nabla}$

$\omega^2 r/g$	HEMISPHERE	TORUS	"MONSTER"	BARGE	CYLINDRICAL FLOAT
0.1	-	-	1.0	0.50	-
0.2	0.17	0.50	3.0	1.00	0.60
0.3	0.25	0.87	3.4	1.45	0.78
0.4	0.30	1.22	3.5	1.82	0.91
0.6	0.36	1.65	3.6	2.25	1.03
0.8	0.40	1.85	4.0	2.08	1.07
1.0	0.41	1.93	-	1.87	1.08
1.2	0.41	1.70	-	1.73	-
1.4	0.42	1.10	-	1.68	-
1.6	0.43	0.90	-	1.69	-
1.8	-	0.90	-	1.70	-
Estimated Accuracy	±0.1	±0.25	±0.5	±0.1	±0.1

TABLE 1c

$k_{xx} = \frac{\text{added mass force in x-direction due to x-motion}}{\omega^2 x \rho \nabla}$

$\omega^2 r/g$	HEMISPHERE	TORUS	"MONSTER"	BARGE (SURGE)	BARGE (SWAY)	CYLINDRICAL FLOAT
0.1	-	data not sufficiently accurate (k_{xx} small)	data not sufficiently accurate (k_{xx} small)	data not sufficiently accurate (k_{xx} small)	data not sufficiently accurate (k_{xx} small)	0.82
0.2	0.34					0.91
0.3	0.39					0.99
0.4	0.42					1.05
0.6	0.47					1.07
0.8	0.47					1.00
1.0	0.42					0.95
1.2	0.34					-
1.4	0.29					-
Estimated Accuracy	±0.05	-	-	-	-	±0.05

TABLE 1d

$k_{xx} = \frac{\text{out-of-phase force in x-direction due to x-motion}}{\omega^2 x \rho \nabla}$

$\omega^2 r/g$	HEMISPHERE	TORUS	"MONSTER"	BARGE (SURGE)	BARGE (SWAY)	CYLINDRICAL FLOAT
0.1	-	0.51	0.80	-	-	0.30
0.2	-	0.51	0.80	0.17	0.20	0.30
0.3	-	0.52	0.80	0.18	0.20	0.33
0.4	-	0.53	0.80	0.19	0.20	0.40
0.6	0.05	0.54	0.80	0.21	0.23	0.53
0.8	0.17	0.55	0.80	0.22	0.27	0.61
1.0	0.29	0.57	0.80	0.24	0.29	0.61
1.2	0.35	0.58	0.80	0.26	0.33	-
1.4	0.37	0.60	-	0.28	0.36	-
1.6	-	0.61	-	0.29	0.38	-
1.8	-	0.63	-	0.30	0.40	-
Estimated Accuracy	±0.05	±0.1	±0.2	±0.1	±0.1	±0.05

TABLE 1e

$k_{\theta x} = \frac{\text{added mass pitching moment due to x-motion}}{\omega^2 x \rho \nabla}$

$\omega^2 r/g$	HEMISPHERE	TORUS	"MONSTER"	BARGE (SURGE)	BARGE (SWAY)	CYLINDRICAL FLOAT
0.1	negligible	-	-11.0	-	-	-
0.2		-2.50	-10.9	-1.80	-1.70	+1.11
0.3		-2.50	-10.8	-1.79	-1.70	1.12
0.4		-2.50	-10.9	-1.77	-1.70	1.13
0.6		-2.50	-11.1	-1.76	-1.70	1.15
0.8		-2.48	-11.2	-1.74	-1.69	1.15
1.0		-2.42	-11.1	-1.70	-1.67	1.14
1.2		-2.36	-	-1.66	-1.65	-
1.4		-2.31	-	-1.63	-1.62	-
1.6		-2.27	-	-1.61	-1.58	-
1.8		-2.24	-	-1.59	-1.54	-
Estimated Accuracy	-	±0.2	±1.0	±0.2	±0.2	±0.15

TABLE 1f

$h_{\theta x} = \frac{\text{out-of-phase pitching moment due to x-motion}}{\omega^2 x r \rho \nabla}$

$\omega^2 r/g$	HEMISPHERE	TORUS	"MONSTER"	BARGE (SURGE)	BARGE (SWAY)	CYLINDRICAL FLOAT
0.1	negligible	-	-1.15	-	-	-
0.2		-0.36	-1.16	0.0	-0.14	+0.15
0.3		-0.38	-1.16	-0.03	-0.17	0.18
0.4		-0.40	-1.17	-0.09	-0.19	0.21
0.6		-0.48	-1.20	-0.22	-0.23	0.29
0.8		-0.56	-1.22	-0.31	-0.27	0.34
1.0		-0.60	-1.22	-0.36	-0.29	0.31
1.2		-0.61	-	-0.40	-0.31	-
1.4		-0.60	-	-0.42	-0.33	-
1.6		-0.59	-	-0.42	-0.35	-
1.8		-0.57	-	-0.43	-0.36	-
Estimated Accuracy	-	±0.15	±0.25	±0.1	±0.1	±0.1

TABLE 1g

$k_{x\theta} = \frac{\text{added mass force in x-direction due to }\theta\text{-motion}}{\omega^2 \theta r \rho \nabla}$

$\omega^2 r/g$	HEMISPHERE	TORUS	"MONSTER"	BARGE (PITCH)	BARGE (ROLL)	CYLINDRICAL FLOAT
0.1	negligible	data not sufficiently accurate ($k_{x\theta}$ small)	-	-	-	1.06
0.2			0.032	-0.020	-	1.29
0.3			0.032	-0.013	-0.125	1.40
0.4			0.033	-0.010	-0.092	1.48
0.6			0.033	-0.006	-0.054	1.51
0.8			0.033	-0.003	-0.033	1.41
1.0			-	-0.002	-0.017	1.34
1.2			-	-0.001	-0.005	-
1.4			-	-0.000	+0.006	-
1.6			-	-0.000	+0.015	-
Estimated Accuracy			±0.01	±0.005	±0.005	±0.07

TABLE 1h

$h_{x\theta} = \frac{\text{out-of-phase force in x-direction due to }\theta\text{-motion}}{\omega^2 \theta r \rho \nabla}$

$\omega^2 r/g$	HEMISPHERE	TORUS	"MONSTER"	BARGE (PITCH)	BARGE (ROLL)	CYLINDRICAL FLOAT
0.1	negligible	data not sufficiently accurate ($h_{x\theta}$ small)	-0.063	-	-	0.424
0.2			-0.047	-0.048	-0.033	0.424
0.3			-0.032	-0.039	-0.029	0.466
0.4			-0.025	-0.033	-0.027	0.565
0.6			-0.018	-0.028	-0.027	0.750
0.8			-0.019	-0.030	-0.031	0.862
1.0			-	-0.042	-0.037	0.862
1.2			-	-0.057	-0.045	-
1.4			-	-0.073	-0.054	-
1.6			-	-0.091	-0.063	-
Estimated Accuracy			±0.02	±0.01	±0.01	±0.07

TABLE 1i

$k_{\theta\theta} = \frac{\text{added mass moment in }\theta\text{-direction due to }\theta\text{-motion}}{\omega^2 \theta r^2 \rho \nabla}$

$\omega^2 r/g$	HEMISPHERE	TORUS	"MONSTER"	BARGE (PITCH)	BARGE (ROLL)	CYLINDRICAL FLOAT
0.1	negligible	-	0.220	-	-	-
0.2		0.052	0.134	0.069	0.0087	1.11
0.3		0.042	0.108	0.062	0.0126	1.12
0.4		0.035	0.090	0.057	0.0145	1.13
0.6		0.028	0.067	0.052	0.0168	1.15
0.8		0.024	0.057	0.049	0.0178	1.15
1.0		0.021	0.051	0.048	0.0170	1.14
1.2		0.020	-	0.047	0.0155	-
1.4		0.019	-	0.044	0.0139	-
1.6		-	-	0.041	0.0122	-
Estimated Accuracy		±0.005	±0.025	±0.01	±0.003	±0.15

TABLE 1j

$h = \frac{\text{out-of-phase moment in }\theta\text{-direction due to }\theta\text{-motion}}{\omega^2 \theta\ {}^2 \rho \nabla}$

$\omega^2 r/g$	HEMISPHERE	TORUS	"MONSTER"	BARGE (PITCH)	BARGE (ROLL)	CYLINDRICAL FLOAT
0.1	negligible	-	0.055	-	-	-
0.2		-0.008	0.043	-0.070	-0.0138	+0.15
0.3		+0.001	0.040	-0.038	-0.0081	0.18
0.4		0.006	0.040	-0.020	-0.0048	0.21
0.6		0.012	0.048	-	-	0.29
0.8		0.015	0.063	0.012	0.0032	0.34
1.0		0.016	-	0.015	0.0055	0.31
1.2		0.017	-	0.014	0.0071	-
1.4		0.017	-	0.014	0.0083	-
1.6		-	-	0.016	0.0088	-
1.8		-	-	-	-	-
Estimated Accuracy		±0.004	±0.02	±0.003	±0.002	±0.10

Definitions

z = amplitude of forced heave motion

x = amplitude of forced surge motion

θ = amplitude of forced pitch motion

ω = frequency of harmonic motion

∇ = displaced volume of model

ρ = fluid mass density

r = radius of hemisphere with same displacement as model, i.e. $r/\nabla^{1/3} = (3/2\pi)^{1/3} = 0.781$

Frequency parameter = $\omega^2 r/g$, g = acceleration of gravity

in the body-fixed axes. The hydrodynamic forces and moment can be considered as composed of a buoyant or static component, an out-of-phase or damping component, and an inertial component. The vertical force for this mode of motion is negligibly small and, as in the case of horizontal motion, it is possible to detect a second harmonic hydrodynamic force. Tests of the hemisphere were not conducted for this mode of motion since the resulting hydrodynamic forces and moments would be too small to resolve. The coefficients for this mode of motion are given in Table 1g–j; $k_{\theta\theta}$ is the added

Table 2 Wave-exciting forces on restrained models

$\omega^2 r/g$	B_z	γ_z,deg	B_x	γ_x,deg	B_θ	γ_θ,deg
TABLE 2a HEMISPHERE MODEL						
0.0	1.500	0	0	0	0	-
0.2	1.200	-6	0.280	-87	"	-
0.4	0.952	-22	0.480	-94.5	"	-
0.6	0.847	-35	0.701	-96.5	"	-
0.8	0.750	-42	0.860	-98	"	-
1.0	0.619	-48.5	0.874	-100	"	-
1.2	0.489	-58	0.736	-104	"	-
1.4	0.404	-70	0.652	-118	"	-
1.6	0.391	-82.5	0.708	-123	"	-
1.8	0.463	-94.5	0.802	-128	"	-
Estimated Accuracy	±.03	±5	±.05	±5		
TABLE 2b TORUS MODEL						
0	3.08	0	0	-	0	-
0.2	2.31	-9	0,31	-91	0.443	80
0.4	1.88	-31	0.45	-105	0.720	66
0.6	1.91	-48	0.74	-107	1.010	66
0.8	1.84	-57	0.77	-108	1.100	66
1.0	1.66	-55	0.63	-119	0.917	58
1.2	1.46	-42	0.60	-139	0.754	41
1.4	1.28	?	0.60	-163	0.714	20
Estimated Accuracy	±.05	±5	±0.25	±5	±.03	±5
TABLE 2c "MONSTER BUOY" MODEL						
0	4.75	0	0	-	0	-
0.1	4.19	-13	0.187	-84	0.451	63
0.2	3.36	-26	0.315	-96	0.699	69
0.3	2.99	-37	0.446	-104	0.962	74
0.4	2.97	-46	0.544	-112	1.260	79
0.5	2.78	-54	0.587	-119	1.405	83
0.6	2.41	-63	0.604	-126	1.390	81
0.7	1.95	-76	0.602	-134	1.275	73
0.8	1.70	-88	0.610	-142	1.170	62
0.9	1.70	-101	0.635	-150	1.220	48
1.0	1.87	-113	0.685	-158	1.400	35
Estimated Accuracy	±.1	±5	±.03	±5	±.05	+-5

$\omega^2 r/g$	B_z	γ_z,deg	B_x	γ_x,deg	B_θ	γ_θ,deg
TABLE 2d BARGE MODEL IN HEAD SEAS						
0	4.170	0	0	-	0	-
0.2	2.820	-6	0.185	-87	0.752	89
0.4	2.080	-33	0.246	-108	1.160	69
0.6	1.990	-53	0.300	-126	1.500	57
0.8	1.770	-68	0.320	-144	1.690	50
1.0	1.386	-84	0.320	-161	1.605	42
1.2	0.985	-99	0.315	-178.5	1.450	33
1.4	0.847	-115	0.305	-195	1.300	24
1.6	0.847	-130	0.293	-210	1.225	15
Estimated Accuracy	±.075	±5	±.015	±10	±.04	±5
TABLE 2e BARGE MODEL IN BEAM SEAS						
0	4.170	0	0	-	0	-
0.2	2.86	-8	0.193	-94	0.367	72
0.4	2.10	-36	0.308	-111	0.579	66
0.6	1.96	-65	0.408	-122	0.835	60
0.8	2.10	-74	0.500	-129	1.060	56
1.0	2.02	-77	0.555	-136	1.140	52
1.2	.1.79	-81	0.494	-141	1.010	47
1.4	1.62	-85	0.458	-148	0.920	41
1.6	1.73	-88	0.550	-155	1.055	36
Estimated Accuracy	±.075	±5	±.015	±10	±.04	±5
TABLE 2f CYLINDRICAL FLOAT MODEL WITH DAMPING PLATE						
0	0.378	0	-	-	0	-
0.1	0.223	0	0.019	-100	0.068	-100
0.2	0.142	-6	0.034	-98	0.076	-90
0.3	0.079	-22	0.044	-96	0.071	-80
0.4	0.032	-56	0.052	-93	0.062	-65
0.5	0.006	-145	0.059	-90	0.050	-50
0.6	0.020	-205	0.065	-86	0.041	-23
0.7	0.032	-212	0.069	-82	0.036	+17
0.8	0.039	-215	0.073	-78	0.033	+95
0.9	0.040	-215	0.077	-73	-	-
1.0	0.040	-215	0.080	-68	-	-
Estimated Accuracy	±.01	±5	±.005	±5	±.01	±5

Definitions

$B_z e^{i\gamma_z} = \frac{\text{Heave Force}}{\text{Wave Motion}} \cdot \frac{r}{\rho g \nabla}$, γ_z = phase between heave force maximum and wave trough passage

$B_x e^{i\gamma_x} = \frac{\text{Surge Force}}{\text{Wave Motion}} \cdot \frac{r}{\rho g \nabla}$, γ_x = phase between surge force maximum and wave trough passage

$B_\theta e^{i\gamma_\theta} = \frac{\text{Pitch Moment}}{\text{Wave Motion}} \frac{r}{\rho g \nabla}$, γ_θ = phase between pitch moment maximum and wave trough passage

where:

ω = frequency of harmonic wave motion

∇ = displaced volume of model

ρ = fluid mass density

r = radius of hemisphere with same displacement as model, i.e., $r/\nabla^{1/3} = (3/2\pi)^{1/3} = 0.781$

g = acceleration of gravity

moment of inertia coefficient for the inertial moment, $h_{\theta\theta}$ is the damping coefficient for the out-of-phase moment, $k_{x\theta}$ is the added mass coefficient for the x-directed inertial force, and $h_{x\theta}$ is the damping coefficient for the x-directed out-of-phase force, all due to x-motion.

Wave Tests

For the case of the models in waves, because there is no motion, there will be no tare corrections for the force. Since the flowfield of the exciting waves has gradients of the pressure, fluid velocities, and accelerations, it is not a simple, or a necessary, task to interpret the measured force in terms of buoyant, inertial, and damping components. A system of presentation has been adopted where the exciting force amplitude is nondimensionalized with respect to the model displacement and wave elevation. The phase between the maximum of the force component and the passage of the wave trough is reported. This manner of presentation is convenient for use in the equations of motion, such as Eqs. (7).

The results of the tests are shown in Table 2, where the various coefficients are defined. Test results for all components of the exciting force for a single model are given in one section. A comparison of the measured heave force on the half-immersed sphere model with theoretical results for this case is given in the Discussion.

Discussion

There is not a great amount of information available in published literature concerning the hydrodynamic forces acting on three-dimensional buoy-like bodies in waves. Experimental data for pilings, breakwaters, and a submerged sphere have been reviewed by Dean and Harleman.[10] Procedures have been developed for the study of motions of ships progressing at various headings in waves, as discussed by Korvin-Kroukovsky[3] and others, but these mainly deal with strip theories or slenderbody theories and correlation with experiments for shiplike forms.

Analysis of hydrodynamic forces acting on buoy-like shapes in waves, even for zero speed, is seriously complicated by the need to properly account for body geometry within a mathematically complex framework. Only for fairly simple shapes such as spheres, ellipsoids, and elliptical disks has the analysis been carried out.

Havelock[11] has analyzed the waves and hydrodynamic forces acting on a half-immersed sphere making periodic heaving oscillations. This shape is also studied in a more general paper by Kim,[12] which also covers ellipsoids and treats heave, surge and pitch oscillations. A comparison of the added mass and damping coefficients according to these analyses and the present test results is given in Fig. 6. The shaded bands indicate the range of values for added mass and damping coefficients obtained by Cumming[13] from experiments with a vertically oscillating hemisphere. The scatter in Cumming's results, which were obtained both from direct force measurements and from integration of pressure measurements, is rather great and may be interpreted as indicating the difficulty of obtaining satisfactory measurements. The analysis and the experiments can be said to agree qualitatively, but quantitative agreement is only approximate.

A comparison of the added mass and damping coefficients for vertical motion of the "monster" buoy model with a theory, by Kim,[14] for a flat circular disk, is shown in Fig. 7. The coefficients used here are defined with the cube of the

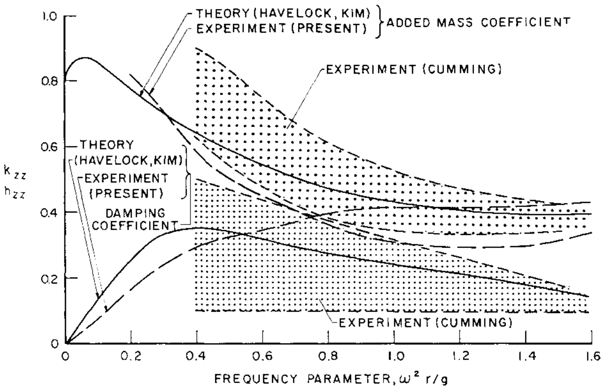


Fig. 6 Comparison of experiment and theory for hemisphere heaving in free surface.

disk radius as a significant volume, rather than the displaced volume (which is zero for a disk), and the equivalent disk radius of the "monster" buoy is taken as its maximum value (4.8 in.) rather than the value at the average waterline (4.56 in.). The theory and experiments again agree in a qualitative sense, at least. No further comparisons of these experimental results for heaving motion can be made, either with theoretical or other experimental work.

For surging motion, it is possible to compare the hemisphere model results with Kim's[12] theoretical work, as shown in Fig. 8. Again, the results agree qualitatively. The side force data for the models with shallow draft (torus, monster, and barge) was, as might be expected, fairly small, and the inertial part could not be resolved with sufficient accuracy to report.

The inertial components of the pitch moment coefficients associated with the surging motion ($k_{\theta x}$) of the shallow draft models exhibit a negative sign which is at first consideration unusual. This must be associated with the distribution of pressures over the bottom, rather than with the horizontal component of the pressure distribution force, since the horizontal component must give a positive inertial moment. A similar finding for two-dimensional cylinders of shallow draft has been shown by Vugts,[15] both theoretically and experimentally.

The vertical wave exciting force for the hemisphere has been compared with a theoretical computation by Barakat[16] in Fig. 9. The experimental and theoretical results for the force amplitude differ in the low and high frequency range, but agree well in the important middle frequency range. The wave force measurements are considered to be fairly accurate, in general (see Table 2), and the experimental determination in the range of frequency parameter of 1.6–1.8 is well established.

It may be noted that, in several instances, the estimated accuracy of the data is not as good as might be hoped for. This is partly associated with the inherent difficulties in

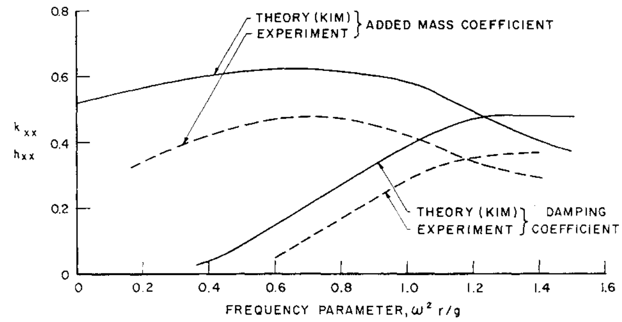


Fig. 8 Comparison of experiment and theory for hemisphere surging in free surface.

making such measurements which depend upon careful matching of apparatus and models, as well as careful attention to test technique and data analysis procedures. It appears that the monster buoy model, in particular, was not sufficiently large for these tests, and a scale ratio of about $\frac{1}{35}$ would have produced more satisfactory results.

Additional data concerning forces acting on various vertical cylindrical floats with horizontal damping plates on the bottom have been reported by Tseng and Altmann.[17] These valuable results are not compared with results obtained from the present tests because their models had much higher ratios of draft to diameter. Data were analyzed somewhat differently also, especially for the out-of-phase component which was assumed to be proportional to the square of the maximum velocity as distinct from our representation of linear (viscous) damping. No tests were conducted for pitch oscillations, but procedures for approximating the resulting forces are given.

Some data for wave excitation forces acting on models composed of an immersed sphere connected to a vertical, surface-piercing cylinder have been reported by Motora and Koyama.[18] These related results are of interest.

The analysis of the data, as mentioned previously, under Test Procedure, is quite laborious. There are techniques available for obtaining the Fourier coefficients of the harmonic output of the force transducers in tests such as these. These techniques have been described by Gertler[19] and Zunderdorp and Buitenhek[20] and were applied in the tests of Tseng and Altmann.[17] Some information about the regularity of the periodic forces and the importance of higher harmonics is usually overlooked in applying these techniques. However, the avoidance of tedious oscillogram analysis makes the use of these data-gathering procedures desirable. A new procedure has been developed for recently conducted experiments at Davidson Laboratory in which a computer controlled (PDP-8) analog-to-digital converter samples the signals for forces, moments, waves and/or motions and the Fourier coefficients are obtained by integrating over approximately ten cycles of oscillations or waves. The motion signal is derived

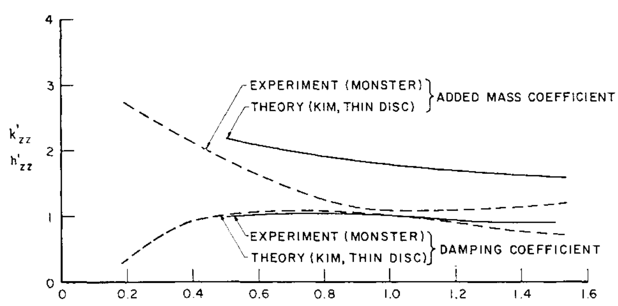


Fig. 7 Comparison of experiment and theory for thin disk heaving in free surface.

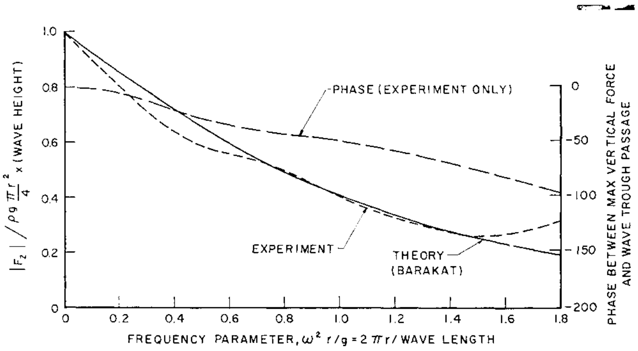


Fig. 9 Comparison of experiment and theory for vertical force on hemisphere in waves.

from the output of a cosine potentiometer attached to the drive shaft of the oscillator and phases of force and moment signals are referred to either wave or motion signals. Also, there is a place for the use of the free oscillation technique in studies such as this and it is suggested that this procedure be applied in addition to the forced oscillation procedure, for several cases.

Conclusions

Test results for hydrodynamic forces acting on several models of typical buoy shapes have been presented and discussed. The forces measured were lift, drag, and pitch moment under conditions of harmonic motion in heave, surge, and pitch, and for the models held fixed with harmonic surface waves passing by. These results may be used together with the equations of motion to evaluate the motions of these bodies for arbitrary mass distributions. Other forces, such as gyroscopic forces associated with rotating machinery, or mooring cable forces, may also be incorporated into such an analysis.

Comparisons of some of these results with those of available theory and other experiments indicate a qualitative agreement. The estimated accuracies of some of the coefficients listed in Table 2 are not very good, so that the accuracy of the values of corresponding forces is doubtful. The usefulness of these results for further application will depend very much on the nature of application and on individual judgment. Some brief comments were made concerning improvements or alternatives in experimental technique which should be incorporated in future experimental work of this kind.

Recommendations

1) The usefulness of the present data should be tested by evaluating the motions of various buoy configurations in waves and comparing the results with observed motions wherever possible. 2) A theory should be developed for evaluating the hydrodynamic forces acting on arbitrary float shapes. 3) Correlation of theoretical and experimental results should always be performed wherever possible.

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